

PROTON DECAY, BLACK HOLES, and LARGE EXTRA DIMENSIONS

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Abstract

We consider proton decay in theories that contain large extra dimensions. If virtual black hole states are allowed by the theory, as is generally the case, then proton decay can proceed via virtual black holes. The experimental limits on the proton lifetime place strong constraints on the quantum gravity scale M_{qg} (the effective Planck mass). For most theories, this implies a lower bound of $M_{\text{qg}} > 10^{16}$ GeV. The corresponding bound on the size of large extra dimensions is $\ell < 10^{6/n} \times 10^{-30}$ cm, where n is the number of such dimensions. Regrettably, for most theories this limit rules out the possibility of observing large extra dimensions at accelerators or in millimeter scale gravity experiments. Conversely, proton decay could be dominated by virtual black holes, providing an experimental probe to study stringy quantum gravity physics.

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I. INTRODUCTION

In conventional versions of string theory (M theory), the string energy scale, the Planck mass, and the unification scale are roughly comparable and are relatively close to the standard value of the Planck mass, $M_{\text{pl}} \sim 10^{19}$ GeV. Recently, however, the possibility of a much smaller string scale and large extra dimensions – perhaps large enough to be observable in particle accelerator and gravity experiments – has sparked a great deal of interest [1–11]. As is well known, one of the many constraints on such theories with large extra dimensions is the rate of proton decay. The current experimental limit on the proton lifetime depends on the particular decay mode under study [12], but for the most interesting channels the bound is approximately $\tau_P > 10^{33}$ yr [13]. In the context of theories with large extra dimensions, the existing theoretical literature includes various papers which present mechanisms to suppress “ordinary” GUT scale proton decay, i.e., decay driven by intermediate bosons that mediate baryon number non-conservation. In this case, one can invent extra symmetries to get rid of those unwanted processes and thereby suppress the proton decay rate below the experimental limit [4–7]. In this present paper (see also Refs. [4,11]), we address the idea of virtual black holes acting as the intermediate particles as they do in gravitationally induced proton decay [14–18]. In particular, we find lower bounds on the quantum gravity scale M_{qg} , upper bounds on the size ℓ of the extra dimensions, and limits from higher order proton decay processes with $\Delta B > 1$.

This topic has some urgency: If the relevant scale for quantum gravity is as low as 1 TeV, for example, then quantum gravity effects could in principle be observed in existing (or upcoming) accelerators, or as devia-

tions from Newtonian gravity. One of the proposed experimental signatures of quantum gravity would be virtual black holes [8,19]. Unfortunately, the same virtual black holes that could be observed could also drive proton decay at a rapid rate, larger than allowed by existing experimental bounds on the proton lifetime. As a result, gravitationally induced proton decay must be highly suppressed in any theory of quantum gravity that accommodates low energy scales and thereby allows large extra dimensions; possible mechanisms have been suggested to provide such suppression and remain under study [4,5,6,11]. However, in a viable theory such a suppression must allow a way to generate a baryon asymmetry and neutrino masses; these can be linked by $B - L$ conservation in many theories. So far, that has not been achieved to our knowledge. The burden of proof must be, to a large extent, placed on any comprehensive new approach, so that it demonstrates that it is not inconsistent with these constraints. The arguments of this paper thus suggest interesting constraints on approaches to quantum gravity that allow for virtual black hole states at low energy scales, such as recent approaches invoking millimeter or TeV size dimensions.

For another new approach, the Randall-Sundrum case [20,21], the Standard Model (SM) masses appear at about the TeV energy scale while the original mass parameter of the theory can remain near the (old) Planck scale M_{pl} . The energy scale for quantum gravity effects and virtual black holes may remain large ($\sim M_{\text{pl}}$) and hence this class of theories might evade the bounds of this paper. However, in most versions of Randall-Sundrum ideas where Kaluza-Klein (KK) states are at the TeV scale, and where black hole sizes are determined by the KK masses, our arguments should apply.

II. PROTON DECAY IN 4 DIMENSIONS

Let's first review the picture of gravitationally induced proton decay using three spatial dimensions and the traditional value of the Planck mass ($M_{\text{pl}} \sim 10^{19}$ GeV). In any quantum theory, one expects to find vacuum fluctuations associated with the fundamental excitations of the theory. In electrodynamics, for example, electron-positron pairs can form directly out of the vacuum (for a short time). Such processes can be observed indirectly by many quantum phenomena (e.g., in the Casmir effect). In general, however, one must include in the vacuum processes all possible excitations of the theory, e.g., the production of proton-antiproton pairs, or even monopole-antimonopole pairs. These processes are generally highly suppressed relative to the electron-positron amplitudes by virtue of their correspondingly large masses. In gravitation, one therefore expects not only to find virtual gravitons playing a role, but also virtual black holes. Although present uncertainties in quantum gravity theory prevent reliable calculations, even in the context of string theory, we can use a semi-classical calculation as a starting point to study such phenomena.

The standard arguments for virtual black holes lead to the idea that spacetime must be filled with tiny Planck mass black holes with a density of roughly one per Planck volume [22,23]. These microscopic virtual black holes then live roughly for one Planck time. This picture of the spacetime vacuum is often called the *spacetime foam* (a generalization of this argument for higher dimensions is sketched in the Appendix). Since black holes and the Standard Model itself are presumed not to conserve baryon number, these virtual black holes contribute to the rate of proton decay through their gravitational interaction. In this setting, a proton is considered to be a

hollow sphere of radius $R \sim m_P^{-1} \sim 10^{-13}$ cm that contains three (valence) quarks. Suppose that two of these quarks fall into the same black hole at the same time (the quarks must be pointlike compared to the black hole scale for this argument to hold). Since the black hole will evaporate predominantly into the lightest particles consistent with conservation of charge, energy, and angular momentum, this process effectively converts the quarks into other particles; only rarely will the same quarks come out that originally went into the black hole. The output particles will often be antiquarks and leptons, and hence baryon number conservation is generally violated. In other words, quantum gravity introduces an effective interaction leading to many final states, including processes of the form

$$q + q \rightarrow \bar{q} + \ell, \quad (1)$$

where the final state can include any number additional particles (gravitons, gluons, photons, neutrinos, etc.) and where the resulting antiquark will generally hadronize (e.g., to π^0). These interactions can be regarded as four-Fermi interactions whose coupling strength is determined by the Planck mass. Such processes are mediated by black holes and can violate conservation of baryon number. Notice, however, that these processes cannot be mediated by gravitons alone because such interactions conserve both electric charge and baryon number. Notice also that we are implicitly adopting the Hawking picture of black hole evaporation (which is information non-preserving).

The probability of two quarks being within one Planck length ($\ell_{\text{pl}} \sim 10^{-33}$ cm) of each other inside a proton is about $(m_P/M_{\text{pl}})^3 \sim 10^{-57}$. This value represents the probability per proton crossing time $\tau \sim m_P^{-1} \sim 10^{-31}$ yr, if we assume that the particles move at the speed of light. In order for an interaction to take place (such as equation [1]), a virtual black holes must

be present at the same time that the two quarks are sufficiently near each other. Including this effect reduces the overall interaction probability by an additional factor of m_P/M_{pl} . Converting these results into a time scale for proton decay [14–18], we find an estimated proton lifetime of

$$\tau_P \sim m_P^{-1} \left(\frac{M_{\text{pl}}}{m_P} \right)^4 \sim 10^{45} \text{ yr} . \quad (2)$$

For comparison, if the proton is unstable through some process operating at the (nonsupersymmetric) unification (GUT) scale M_X [12], the corresponding time scale for proton decay becomes

$$\tau_P \sim 10^{30} \text{ yr} \left(\frac{M_X}{10^{15} \text{ GeV}} \right)^4 . \quad (3)$$

Thus, the proton lifetime expected from virtual black hole processes (equation [2]) is the same as that for GUT scale processes (equation [3]) in the limit that the unification scale M_X approaches the Planck scale M_{pl} and the coupling becomes of order unity. For completeness, we note that in supersymmetric theories the unification scale can be somewhat higher than in grand unified theories without SUSY; in this case, the proton lifetime can be as long as $\tau_P = 10^{33} - 10^{34} \text{ yr}$, consistent with current experimental limits.

Most versions of string theory contain black hole states with masses comparable to the Planck mass (the quantum gravity or string scale). These black holes play the role of the X -boson in proton decay, independent of any specific argument about virtual black holes or spacetime foam. Furthermore, we have no reason not to believe the Hawking formula for the entropy of stringy black holes or their temperature; as a result, the density of states formula implicitly used here should be correct in string theory. One remaining controversy is whether or not the black holes genuinely lose information. Most particle physicists say they do not, whereas most relativists say they

do. At present, we simply do not know. Notice that if only $B-L$ is conserved in string theory, then protons will decay with the rate derived here.

One still might be concerned about suppression of the proton decay rate due to violation of global conservation laws or information loss. Because of the t'Hooft anomaly, however, baryon number is not conserved even in the SM, although electric charge and possibly $B-L$ are. The decay described by equation (1), e.g., conserves these latter quantum numbers. Channels such as this decay, that could conserve global quantum numbers and perhaps even circumvent information loss issues, can dominate [10]. As a result, there is no compelling reason to expect large suppressions of the decay channel.

In equation (1), the virtual black holes mediating the interaction appear to act like local quantum objects and thus one might be concerned that the interaction could be gauged away, much like what is done to remove unwanted interactions involving X -bosons. Unlike local quantum particles, however, the virtual black holes in quantum gravity processes are solitonic objects – they are less likely to be gauged away because they are extended. In order to suppress proton decay mediated by virtual black holes, one would need to make [5,6] baryon number (or an equivalent matter parity) into a local charge via an exact gauged discrete symmetry that is fully respected by the true vacuum of the theory. Even then, proton decay is often only suppressed up to some (possibly quite large) order in the effective operators. Such a suppression requires very special arrangements of chiral fermions or other aspects of the theory. For example, as pointed out by Kakushadze [6], a \mathbf{Z}_3 “Generalized Baryon Parity” cannot accommodate right-handed neutrinos without adding additional matter because of anomaly cancellation constraints. It is not an accident that the neutrino sector affects the efforts to enforce such suppressions; B and L conservation are related in many models

by conservation of $B - L$. To allow majorana neutrino masses, L cannot be conserved. As we argue below, however, because theories with large extra dimensions tend to allow rapid proton decay via virtual black holes, any viable such theory must include a strong suppression mechanism.

Some mechanisms have been suggested [11,27] to avoid the proton decay problem in the presence of large extra dimensions. They typically have in common the need for a new scale, one that is not given by scale of quantum gravity and is not determined by conventional (known) scales such as the electroweak or supersymmetry breaking scales. In Ref. [11], for example, quarks and leptons are embedded in domain walls separated by a new scale of order 50 times the distance (or more) one would expect if the wall parameters were determined by the quantum gravity scale; if the domain wall thickness was specified only by the quantum gravity scale, then virtual black hole states would be large enough to bridge the gap and drive proton decay. If the physical origin of such a new scale can be identified, and a reason why a stable separation of quarks and leptons should occur, that would represent important progress. One additional general constraint on such ideas is the need to generate a baryon asymmetry in the early universe (this issue is not addressed in Ref. [11]). Currently, quarks and leptons are separated by imposing the counter-intuitive requirement that their Yukawa couplings are of opposite sign, which is possible but, as yet, unmotivated.

Although it is not yet absolutely proven that baryon number cannot be effectively conserved in the presence of black holes, we find it unlikely for two reasons: **[A]** Astrophysical black holes seem to manifestly violate such a conservation law. Imagine compressing a star containing $N_B \sim 10^{57}$ baryons into a black hole and watching it radiate away. Because the Hawking temperature is low for most of its evaporation time, the black hole radiates primarily

into photons, gravitons, and neutrinos; the temperature becomes hot enough to radiate quarks, protons, or other baryonic particles only after the mass shrinks by 20 orders of magnitude. Thus, for baryon number to be conserved, the theory would have to contain extremely unusual objects with small mass and huge baryon number – the baryon number to mass ratio would be 20 orders of magnitude larger than that of the proton. This case may not be explicitly excluded but it is nonetheless extremely unlikely; it requires the introduction of a new factor of 10^{20} , which is ten million times larger than the dimensionless number ($M_{GUT}/M_{Weak} \sim 10^{13}$) represented by the hierarchy problem. (Notice that the black hole may not actually disappear, just as an electrically charged black hole may not disappear – it remains in a BPS-like configuration; the implications of this possibility remain unclear). **[B]** The observed baryon asymmetry in the universe argues strongly against absolute conservation of baryon number. Unless one posits special initial conditions at the Big Bang, the cosmos had to generate a baryon excess through *some* process that violates conservation of baryon number.

In string theory, a large number of BPS states are known to be extreme black holes [24], and presumably many other massive string states are black holes as well. All of these states can mediate decays as in equation (1). Because so few known experimental tests can probe the string nature of quantum gravity theories, we should turn our argument around: Since string theory is likely to allow proton decay via virtual black holes and since the string scale may be as low as the GUT scale, proton decay modes (such as those explored in this paper) may be a very powerful diagnostic of string theories. Furthermore, the gravitationally induced channels of proton decay should be recognizable in experiments from their observed branching ratios. Thus, proton decay could be a valuable way to study strong gravitational

interactions. This issue should be studied in greater detail in future work.

III. PROTON DECAY WITH LARGE EXTRA DIMENSIONS

We now consider the process of gravitationally induced proton decay in theories with large extra dimensions. For proton decay driven by non-gravitational means – for intermediate particles other than virtual black holes – it is possible to enforce symmetries on the theory to prevent proton decay at overly fast rates [4–7]. In the case of gravity, however, such suppressions are more difficult, particularly when quantum fluctuations are large. Working in worlds with large extra dimensions, Emparan et al. [10] have argued that black hole evaporation occurs mostly on the brane; this result thus strengthens our approach since final states with particles lighter than a proton are not suppressed.

In a theory with large extra dimensions, two effects modify the picture of proton decay outlined above:

[A] The most important modification is that the Planck mass changes. Specifically, the energy scale of virtual black hole processes changes from $M_{\text{pl}} \approx 10^{19}$ GeV to a lower value which we denote here as M_{qg} . Because the new quantum gravity scale M_{qg} is generally lower than both M_{pl} and the GUT scale, this effect acts to reduce the proton lifetime. In particular, the Schwarzschild radius for the virtual black holes is given by $R_S \sim M_{\text{qg}}^{-1}$ [8], which determines the cross section and is much larger than before.

[B] If the number of extra large dimensions is $n > 0$, then the geometry of both the proton and the virtual black holes change. In this context, we let $d \leq n$ denote the number of extra dimensions *that the quarks can propagate through*. In most theories with large extra dimensions, quarks and other SM

particles are confined to the usual 4-dimensional world and cannot freely propagate in the extra dimensions; for most cases, we thus have $d = 0$. In the general case with $d > 0$, the quarks that make up the proton have more dimensions in which to propagate. With more dimensions, the quarks would be less likely to encounter each other and hence this effect increases the proton lifetime. On the other hand, the black holes must be $(4 + n)$ dimensional objects and will necessarily live in the additional dimensions. The black hole interaction cross sections remain of order $R_S^2 \sim M_{\text{qg}}^{-2}$, however, even when interacting with SM particles confined to the usual 4-dimensional spacetime (where we assume strong coupling).

Including the above two modifications in estimating the proton decay rate through virtual black hole processes, we find the proton lifetime

$$\tau_P \sim m_P^{-1} \left(\frac{M_{\text{qg}}}{m_P} \right)^{4+d}. \quad (4)$$

The current experimental bound on the proton lifetime [13] can be written in the form

$$\tau_P > 10^{33} \text{ yr} \equiv m_P^{-1} \left(\frac{\Lambda}{m_P} \right)^4, \quad (5)$$

where we have defined an energy scale $\Lambda \equiv (m_P^5 10^{33} \text{ yr})^{1/4} \approx 1.4 \times 10^{16} \text{ GeV}$. Combining the general expression (4) with the experimental bound (5), we thus obtain a bound on the scale M_{qg} of quantum gravity:

$$M_{\text{qg}} > (m_P^d \Lambda^4)^{1/(4+d)} = 10^{64/(4+d)} \text{ GeV}, \quad (6)$$

where we have used $m_P \sim 1 \text{ GeV}$ and $\Lambda \sim 10^{16} \text{ GeV}$ to evaluate the bound in the second equality. This result (when applicable) constrains the possibility of having a low quantum gravity scale that could be observed in present-day or future accelerators. For the most likely case $d = 0$, where quarks

are confined to our 4-dimensional brane, the quantum gravity scale must be comparable to the (usual) GUT scale, i.e., $M_{\text{qg}} \geq 10^{16}$ GeV. For $d = 1 - 2$, the quantum gravity scale remains quite high. The weakest constraint arises if $d = 7$, which corresponds to the (unlikely) case in which all of the possible extra dimensions are large and the valence quarks within the proton are allowed to propagate freely through all dimensions; in this case, the limit on the quantum gravity scale is $M_{\text{qg}} > 700$ TeV. This scale remains interesting in terms of modifying the hierarchy problems associated with a high quantum gravity scale ($M_{\text{pl}} \sim 10^{19}$ GeV), but unfortunately it remains safely out of experimental reach.

We can also find corresponding bounds on the size scales of the extra dimensions. This size scale ℓ is determined by

$$\ell^n = M_{\text{pl}}^2 M_{\text{qg}}^{-(2+n)}, \quad (7)$$

where n is the number of extra large dimensions (see Ref. [1–11]). For the most likely case with $d = 0$, equation (6) implies the bound

$$\ell < (M_{\text{pl}}/\Lambda)^{2/n} \Lambda^{-1}. \quad (8)$$

As a result, the “large” extra dimensions in such a theory would actually be rather small, $\ell < 10^{6/n} \times 10^{-30}$ cm. These size scales would be impossible to observe in modified gravity experiments.

Other rare decays mediated by virtual black holes, such as $\mu \rightarrow e\gamma$ or neutrino disappearance, may also provide limits on large extra dimensions and low quantum gravity scales. We leave the study of these effects to a future analysis.

Thus far, we have only considered processes which lead to the decay of a single proton, i.e., processes with $\Delta B = 1$. However, many of the

possible suppression mechanisms for proton decay forbid $\Delta B = 1$ decays, but allow larger ΔB . For example, $\Delta B = 2$ can appear in neutron-antineutron transitions. We can immediately generalize the expression for the proton lifetime (eqs. [2 – 3]) for the case of $\Delta B = N$ (see Ref. [28]),

$$\tau_P \sim m_P^{-1} \alpha^{-2N} \left(\frac{M_{\text{qg}}}{m_P} \right)^{4N} \sim 10^{33} \text{yr} \, 10^{64(N-1)} \left(\frac{M_{\text{qg}}}{10^{16} \text{GeV}} \right)^{4N}, \quad (9)$$

where α is the coupling constant and where we have taken the likely case of $d = 0$. For example, if we use the scale $M_{\text{qg}} = 1$ TeV for quantum gravity, the proton lifetime for $N = 2$ is only a few seconds; for $N = 3$, the proton lifetime is only $\tau_P \sim 10^5$ yr. We note that processes with $\Delta B \geq 2$ require the protons to be near each other. Such processes will thus take place in large nuclei (e.g., iron) and in compact stellar objects (e.g., neutron stars) but free protons in interstellar space would not be affected.

We also obtain the corresponding bound on the quantum gravity scale M_{qg} ,

$$M_{\text{qg}} > m_P (\Lambda/m_P)^{1/N} \approx 10^{16/N} \text{GeV}. \quad (10)$$

For the case of $N=2$, for example, our bound becomes $M_{\text{qg}} > 10^8$ GeV; for $N=3$, the bound becomes $M_{\text{qg}} > 2 \times 10^5$ GeV. Thus, higher order proton decay processes (with $\Delta B \geq 2$) also place interesting limits on the quantum gravity scale M_{qg} .

IV. SUMMARY

This paper argues that gravitationally induced proton decay – virtual black hole processes that violate baryon number conservation – should be taken seriously as they imply strong constraints on theories of quantum gravity with large extra dimensions. In particular, this analysis suggests that the

observed absence of proton decay via virtual black holes puts a lower limit on the quantum gravity scale M_{qg} and a corresponding upper limit on the size ℓ of large extra dimensions. In the weakest (and unlikely) case in which quarks propagate in $n = d = 7$ large extra dimensions, the limit is $M_{\text{qg}} > 700$ GeV. This bound rapidly increases to $M_{\text{qg}} > 10^{16}$ GeV for any number n of large extra dimensions, if quarks move only in 3 spatial dimensions ($d = 0$) as is generally required for theories to retain the usual SM physics [1–11,25]. The corresponding bound on the size scale of the extra dimensions is $\ell < 10^{6/n} \times 10^{-30}$ cm. The bounds for $\Delta B > 1$ processes are weaker, but the quantum gravity scale is still highly constrained (eq. [10]).

Because the required interactions with black holes are very general, this limit is robust and will not be affected by the domination of specific decay channels. It could be modified if quark sizes (perhaps set by the string scale) are larger than the Planck size, but this does not occur in most approaches. Our limit may not apply if the generally accepted picture of spacetime foam – every Planck volume of spacetime typically contains a virtual black hole for a Planck time (see the Appendix) – is not valid, or if virtual black hole states are charged under some conserved (including quantum corrections) discrete gauge symmetry (as discussed earlier). Other possible mechanisms to suppress virtual black hole effects have been suggested [11] and should be studied further. Any acceptable mechanism should allow an explanation of the cosmic baryon asymmetry and majorana neutrino masses. We feel that this challenge remains a serious issue.

In general, if this limit turns out to be applicable, the quantum gravity scale must be high enough to remove much of the original motivation for large extra dimensions. Unfortunately, this argument would rule out observable effects at colliders and in millimeter-scale gravity experiments; this issue is

thus of vital importance and must be dealt with more fully than it has been so far. Even with this new constraint, however, the quantum gravity scale M_{qg} could still be somewhat lower than before ($M_{\text{qg}} \sim 10^{16} \text{ GeV} < M_{\text{pl}} \sim 10^{19} \text{ GeV}$), which could help alleviate hierarchy problems. Nonetheless, the signatures of virtual black hole processes might be observable in proton decay experiments, which may eventually provide a powerful experimental probe of quantum gravity and string theories.

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APPENDIX: VIRTUAL BLACK HOLES AND SPACETIME FOAM

In this Appendix, we present a version of the standard argument for virtual black holes filling the vacuum and thereby producing a spacetime foam. In this case, however, we generalize the calculation for higher dimensions. We consider gravity to propagate in $4+n$ dimensions, so that n is the number of large extra dimensions. Notice that n depends on the scale. On sufficiently large spatial scales $n \rightarrow 0$ and we must recover old (4-dimensional) Einstein gravity; in this context we are interested in small spatial scales where

black holes must be $(4+n)$ -dimensional. Gravity is controlled by the action [22,23,26]

$$I[g] = -\frac{1}{16\pi G_n} \int_M R (-g)^{1/2} d^{4+n}x - \frac{1}{8\pi G_n} \int_{\partial M} K (-h)^{1/2} d^{3+n}x + C[h], \quad (11)$$

where G_n is the gravitational constant in $(4+n)$ dimensions and R is the Ricci scalar for the metric g_{ab} , which is defined on the spacetime M . The spacetime boundary ∂M has the induced metric h_{ab} . The quantity K is the trace of the second fundamental form on the boundary ∂M and $C[h]$ is a functional of h defined so that the action of Minkowski space vanishes. Extremization of this action for fixed metric on the boundary leads to the Einstein equations for g_{ab} in M .

The path integral for gravity is

$$Z \sim \int \mathcal{D}[g] e^{iI[g]}, \quad (12)$$

where the integral is taken over all metrics g . Our goal is to investigate how black holes contribute to any amplitude in quantum gravity. We first assume that this integral can be approximated by the usual Euclidean continuation. The action for a single Schwarzschild black hole of mass m is then given by

$$I_1 \sim \frac{m^{2+n}}{M_{\text{pl}*}^{2+n}}, \quad (13)$$

where $M_{\text{pl}*}$ is the Planck mass in $(4+n)$ dimensions ($M_{\text{pl}*} \approx M_{\text{qg}}$). This equation should also contain additional geometrical factors, but these are of order unity and convention dependent (depending on how the mass m is defined). Ignoring interactions between the black holes, we find the action for a collection of N black holes to be

$$I_N \sim \frac{Nm^{2+n}}{M_{\text{pl}*}^{2+n}}. \quad (14)$$

In the path integral, the black holes are indistinguishable; each is independent of the others and can be positioned anywhere in space. Since N is undetermined, we can evaluate Z in a box of volume V_n (in $3 + n$ spatial dimensions) to obtain the result

$$Z \sim \int_0^\infty dm \sum_{N=0}^\infty \exp[-4\pi N m^{2+n}/M_{\text{pl}*}^{2+n}] \frac{1}{N!} \left[\frac{V_n}{\ell_{\text{pl}*}^{3+n}} \right]^N. \quad (15)$$

The factor of V_n comes from accounting for the black holes being anywhere in the box, and the factor of $1/N!$ arises from their indistinguishability.

The combination of these results thus defines a probability distribution for having N black holes with mass m . Elementary calculations yield the corresponding expectation values for the number density n_{bh} of black holes and for the black hole mass m_{bh} , i.e.,

$$\langle n_{\text{bh}} \rangle \sim \frac{V_n}{\ell_{\text{pl}*}^{3+n}} \quad \text{and} \quad \langle m_{\text{bh}} \rangle \sim M_{\text{pl}*}, \quad (16)$$

where $\ell_{\text{pl}*}$ is the Planck length in the $(4+n)$ -dimensional spacetime. As before in the case of 4 dimensions, we find that spacetime must be filled with tiny Planck mass black holes with a density of roughly one per Planck volume. These microscopic virtual black holes live roughly for one Planck time. In this generalized case, however, the black holes are $(4+n)$ -dimensional objects and the Planck mass, the Planck volume, and the Planck time are now given by the new (lower) energy scale $M_{\text{pl}*}$ (where $M_{\text{pl}*} \approx M_{\text{qg}}$). This picture of the spacetime vacuum is thus a generalization of the spacetime foam for the case of $(4 + n)$ dimensions.

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